S.E.-Sem IV (Comp & I.T.), AM-IV 23/11/15

Q.P. Code : 5316

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. one is compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Use of stastical Tables permitted.

(4) Figures to the right indicate full marks

1. (a) Evaluate the line integral $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x$ 5

(b) State Cayley-Hamilton theorem & verify the same for $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ 5

(c) The probability density function of a random variable x is

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	2k	0.3	K

Find i) k ii) mean iii) variance 5

(d) Find all the basic solutions to the following problem

Maximize $z = x_1 + 3x_2 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

and $x_1, x_2, x_3 \geq 0$ 5

2. (a) Find the Eigen values and the Eigen vectors of the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ 6

(b) Evaluate $\oint_c \frac{dz}{z^3(z+4)}$ where c is the circle $|z| = 2$ 6

(c) If the heights of 500 students is normally distributed with mean 68 inches and standard deviation of 4 inches, estimate the number of students having heights i) less than 62 inches, ii) between 65 and 71 inches. 8

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3. (a) Calculate the coefficient of correlation from the following data

x	30	33	25	10	33	75	40	85	90	95	65	55
y	68	65	80	85	70	30	55	18	15	10	35	45

6

(b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 100 such samples, how many would you expect to contain 3 defectives i) using the Binomial distribution, ii) Poisson distribution. 6

(c) Show that the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. 8

4. (a) Fit a Poisson distribution to the following data

x	0	1	2	3	4	5	6	7	8
f	56	156	132	92	37	22	4	0	1

6

(b) Solve the following LPP using Simplex method

Maximize $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \le 2$

$x_1 + 4x_3 \le 4$

$x_1, x_2, x_3 \ge 0$ 6

(c) Expand $f(z) = \frac{2}{(z-2)(z-1)}$ in the regions

$i)\ |z| < 1,\ ii)\ 1 < |z| < 2,\ iii)\ |z| > 2$ 8

5. (a) Evaluate using Cauchy's Residue theorem $\oint_C \frac{1-2z}{z(z-1)(z-2)} dz$ where c is $|z| = 1.5$ 6

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(b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6

(c) Solve the following LPP using the Dual Simplex method

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$. 8

6. (a) Solve the following NLPP using Kuhn-Tucker conditions

Maximize $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to $2x_1 + x_2 \leq 5$; and $x_1, x_2 \geq 0$ 6

(b) In an experiment on immunization of cattle from Tuberculosis the following results were obtained

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	155	912
Total	1024	182	1206

Use χ^2 Test to determine the efficacy of vaccine in preventing tuberculosis. 6

(c) i) The regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$

find a) sample means $\bar{x}$ and $\bar{y}$ b) coefficient of correlation between x and y 4

ii) If two independent random samples of sizes 15 & 8 have respectively the means and population standard deviations as

$\bar{x}_1 = 980, \bar{x}_2 = 1012 : \sigma_1 = 75, \sigma_2 = 80$

Test the hypothesis that $\mu_1 = \mu_2$ at 5% level of significance. 4
